

# Study of Nonlinear Behavior and Chaos Phenomena in Power Inverter

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**Abstract**— A study has been done for chaos and bifurcation phenomena in a various kind of single and three phase dc to ac inverter along with nine level stepped wave inverter. The converters are simulated using a software package MATLAB/SIMULINK. It is observed that the inverters moves from periodic operation state to chaotic operation state as bifurcation parameter like input voltage frequency and load to the inverter is changed. The bifurcation pathway includes smooth period-doubling bifurcations as well as border collision bifurcations. Simulated results output voltage and current waveform of single and three phase inverter along with stepped wave inverter are validated by waveforms and FFT spectrum. Inverters with a large number of steps can generate high quality voltage waveforms. The simulation of single phase three level, three phase h-bridge inverter and nine stepped wave inverters is done in software Matlab/Simulink. The FFT spectrums for the outputs are compared and presented to validate the proposed control strategy. This information leads a powerful role for designing practical circuits in power electronics.

**Index Terms**— Bifurcation, dc-ac inverter, Chaos, Stepped wave inverter

## I. INTRODUCTION

All types of power electronics converters may be classified as on linear time-varying dynamical systems because they exhibit a wealth of nonlinear phenomena, including various kinds of bifurcations and chaos. The principal source of non-linearity is the inherent switching action and presence of a potential source of engineering malfunction and failure. In order to avoid these phenomena it is very important to predict and analyze these nonlinear phenomena of a converter. nonlinear components (e.g. the power diodes) and control methods (e.g. pulse-width modulation). These nonlinearities are the occurrence of bifurcations and chaos in power electronics was first reported in the literature by Hamill [1] in 1988. Experimental observations regarding bounded ness, chattering and chaos were also made by Krein and Bass [2] back in 1990. Although these early reports did not contain any rigorous analysis, they provided solid evidence of the importance of studying the complex behavior of power electronics and its possible benefits for practical design. Using an implicit iterative map, the occurrence of period-doublings, sub-harmonics and chaos in a simple buck converter was demonstrated by Hamill [3] using numerical analysis, PSPICE simulation and laboratory measurements.

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The derivation of a closed-form iterative map for the boost converter under a current-mode control scheme was presented later by the same group of researchers [4, 5]. Further work on the bifurcation behavior of the buck converter was investigated by Chakrabarty [6] who specifically studied the bifurcation behavior under variation of a range of circuit parameters including storage inductance, load resistance, output capacitance, etc. In 1996, Fossas and Olivar [7] presented a detailed analytical description of the buck converter dynamics, identifying the topology of its chaotic attractor and studying the regions associated with different system evolutions.

Power electronics is a relatively new and fast-growing area of electronics with wide practical application. In particular, the stability and the bifurcation analysis of the power electronics systems with the pulse-width modulation (ab. PWM) technique has attracted much interest in recent years [8], [9]. Over the past years, a new type of bifurcation phenomena called border-collision bifurcation has been discovered for one or two-dimensional one-parameter families in discrete systems [10], [11].

In reality, bifurcation is to be avoided, but it is also known that designing a system too remote from bifurcation boundaries may degrade performance characteristics. Hence, efforts have been made to study the bifurcation behavior in single, three and nine stepped wave inverters, to show the practical relevance of bifurcations and chaos in power electronics systems. In the first stage simulation are carried out for the single and three phase H-bridge converters, to investigate nonlinearities. In order to provide stable operation for conversion dc in to ac. Input dc can be obtained from battery, fuel cell, Chopper, solar cell etc.

In second stage, the nine stepped inverter is achieved by cascading four single phase H-bridge inverter. This stepped wave inverter has fixed input voltage and a variable output ac voltage as number of H-bridge inverter increase or decrease. The inverters are simulated through a powerful software package MATLAB/SIMULINK. Voltage current waveforms and FFT spectrum are obtained with change in input parameter values to obtain nonlinearities and complex behavior.

## II. BASIC PRINCIPLE

A single phase three stage H-bridge inverter is shown in Fig. 1. The inverter in first stage provides the three state output as well as converts pure dc in to ac. It consist of four Power semiconductor devices which operates as switches ( $T_1, T_2, T_3, T_4$ ), to the output voltage of desired magnitude and frequency. At the same time only two switches

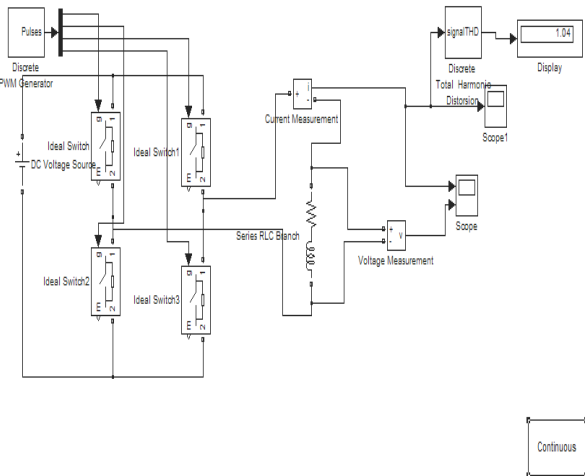
operates and remaining two switches are in off stage. This circuit is easily realized by four switches, a voltage source, an inductor and a resistor. The four switches are named by  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$ . This circuit has the following two conditions.

State A:  $T_1$  and  $T_3$ : ON  
 $T_2$  and  $T_4$ : OFF,

State B:  $T_1$  and  $T_3$ : OFF  
 $T_2$  and  $T_4$ : ON,

**Table I:** Switching States Single Phase Inverter

$T_1$	$T_2$	$T_3$	$T_4$	$V_A$	$V_B$
ON	OFF	ON	OFF	$\frac{V_S}{2}$	$-\frac{V_S}{2}$
OFF	ON	OFF	ON	$+\frac{V_S}{2}$	$+\frac{V_S}{2}$
ON	OFF	ON	OFF	$\frac{V_S}{2}$	$-\frac{V_S}{2}$
OFF	ON	OFF	ON	$+\frac{V_S}{2}$	$+\frac{V_S}{2}$

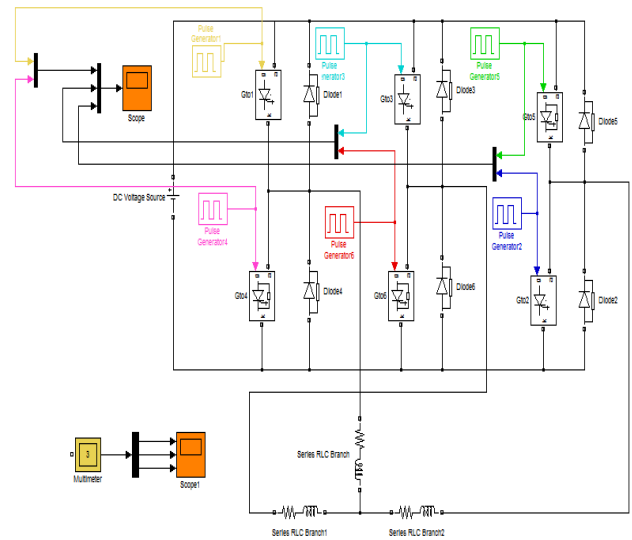


**Fig. 1:** Simulink model of Single phase H-Bridge inverter

H Bridge DC-AC converter operating waveforms is shown in Fig. 5. In one switching cycle, the arriving of clock pulse drives  $T_1$  and  $T_3$  on,  $T_2$  and  $T_4$  off, the inductor current increases; when the inductor current reaches the compensated current reference  $i_{ref}$ ,  $T_1$  and  $T_3$  off,  $T_2$ ,  $T_4$  on, the inductor current begins to decline.

While in three phase inverter number of switches are six ( $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$ ,  $T_5$ ,  $T_6$ ). All these semiconductor switches are conducting current through them only when they are triggered by gate pulses. To obtain gate pulses on gate terminal for conduction of inverter system for single and three phase inverter system firing angles determined through pulse width modulation technique for triggering switches. The basic circuit of three phase full bridge inverter is shown in the following figure 2.

It consist of six power switches associated freewheeling diodes. The switching of switches are periodically in the well proper sequence to produce the desired output waveforms. The arte of switching determines frequency of the inverter.



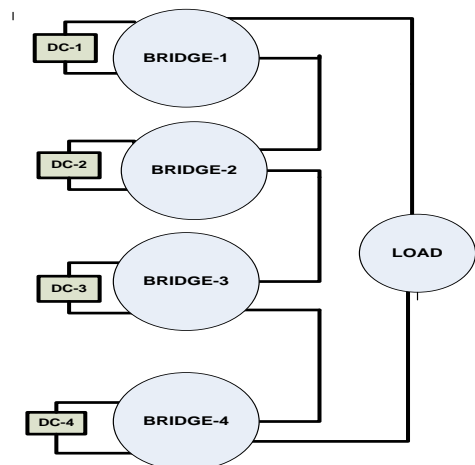
**Fig.2:** Matlab/Simulink model of three phase H-Bridge inverter

**Table II:** Switching States Three Phase Inverter

S.No.	Firing Interval	Turn-on devices	Conducting devices
1	$0^\circ-60^\circ$	$T_1$	$T_5 T_6 T_1$
2	$60^\circ-120^\circ$	$T_2$	$T_6 T_1 T_2$
3	$120^\circ-180^\circ$	$T_3$	$T_1 T_2 T_3$
4	$180^\circ-240^\circ$	$T_4$	$T_2 T_3 T_4$
5	$240^\circ-300^\circ$	$T_5$	$T_3 T_4 T_5$
6	$300^\circ-360^\circ$	$T_6$	$T_4 T_5 T_6$

In second stage, a nine stepped inverter system designed to get desired ac from input dc supply. Four single phase H-bridge inverters cascaded to achieve that nine stepped wave inverter.

Cascaded stepped wave inverters have been proposed for that kind of applications as static var generation, an interface with renewable energy sources, and for battery-based applications. Cascaded inverters are ideal for connecting renewable energy sources with an ac grid, because of the need for separate dc sources, which is the case in applications such as photovoltaics or fuel cells.



**Fig. 3:** Block diagram of nine stepped inverter

From the single phase structure of a cascaded H-bridge inverter as shown in Fig. 1 above, we can make the three level, five level, seven level, nine level, and eleven level inverters without using any type of modulation technique, and by this topology, the number of output-phase voltage steps is defined by  $P = 2Q + 1$ , where 'Q' is the no of levels and 'Q' is the number of DC sources. So, for an example the output phase voltage of nine stepped wave inverter is given by

$$V_{an} = V_{a1} + V_{a2} + V_{a3} + V_{a4} \quad (1)$$

Where  $V_{a1}$ ,  $V_{a2}$ ,  $V_{a3}$ ,  $V_{a4}$  are the voltages across output terminals of h-bridge inverters of nine stepped inverter.

$$V_{an} = V_{a1} + V_{a2} + V_{a3} \quad (2)$$

This output voltage is valid for seven stepped inverter which require three h-bridge inverter. Similarly output voltage of different stepped inverter can be obtain.

Under the assumption that the inductor current is essentially piecewise linear, the dynamics of the controlled current is described by the following map:

$$I_{n+1} = I_n + m_1 T \quad \text{if } I_n \leq I_{ref} - m_1 T \quad (3)$$

$$I_{n+1} = I_{ref} - m_2 t_n \quad \text{if } I_n > I_{ref} - m_1 T \quad (4)$$

Where  $I_n = I_L(nT)$  is the value of the inductor current at the clock instant  $nT$ ;  $m_1$  and  $m_2$  are respectively the magnitudes of the slopes on the increasing and decreasing segment of  $I_L$  and  $t_n$  is the duration of the OFF cycle in the clock in the cycle between  $nT$  and  $(n+1)T$ . Under steady state operation in periodic or chaotic mode, with a constant input voltage  $V_{in}$  and a low ripple output voltage of constant average value  $V_{out}$ , the constants  $m_1$ ,  $m_2$  and  $\alpha$  can be expressed as:

$$m_1 = (V_{in} - V_{out}) / L$$

& 
$$m_2 = V_{out} / L \quad (5)$$

$$\alpha = (m_2 / m_1) = V_{out} / (V_{in} - V_{out}) \quad (6)$$

Here  $\alpha$  is the ratio of slopes magnitude. If  $\alpha > 1$  then, state of operation of the inverter is unstable and circuit has no stable periodic solution. Fig. 2 shows a typical segment of the inductor current,  $I_L$ , of a dc-ac converter under current mode control in the chaotic regime.

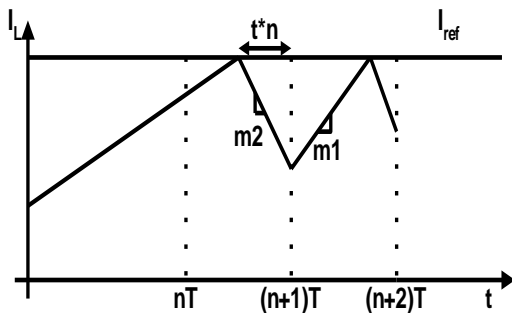


Fig. 4: Typical segment of inductor current for chaotic regime,  $\alpha > 1$

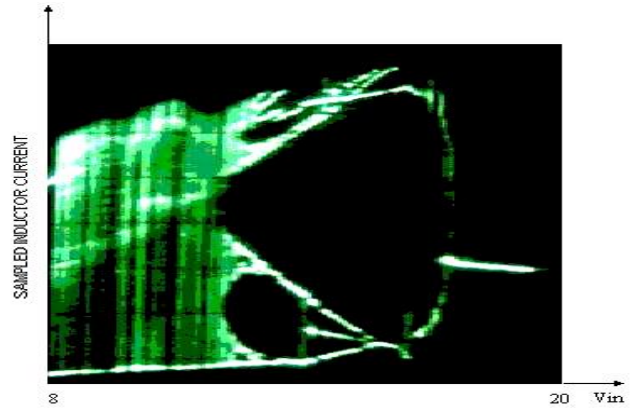


Fig.5: Bifurcation diagram

### III. SIMULATION RESULTS

All inverter circuits of Fig.1, Fig.2 and Fig.12 are simulated on the powerful software package MATLAB/SIMULINK. Fig. 5 shows the Output voltage waveform of single phase h-bridge inverter (period one operation) with input voltage 100V and  $R = 10 \Omega$ ,  $L = 10 \text{mh}$ , Modulation Index = 0.44, frequency = 50Hz. Figure.6 shows enlarge view of output voltage and current waveform when  $V_{in} = 100 \text{V}$ ,  $R = 10 \Omega$ ,  $L = 10 \text{mh}$ , modulation Index = 0.8, frequency = 50Hz which gives stable period-1 operation and first orbit comes into existence. Hence the converter operates in period one operation. Unstable states of voltage and current waveforms

When input voltage = 300V,  $R = 100 \Omega$ ,  $L = 60 \text{mh}$ , modulation index = 0.8, frequency = 50Hz indicated in Fig. 7 & Fig.8. Therefore this orbit subsequently undergoes a border-collision, and around that point another coexisting higher period orbit comes into existence. Top most waveform shows the output current waveform, which reaches in to chaotic state value. Hence the inverter moves to operates in period one operation to chaotic mode operation as bifurcation parameter changes.

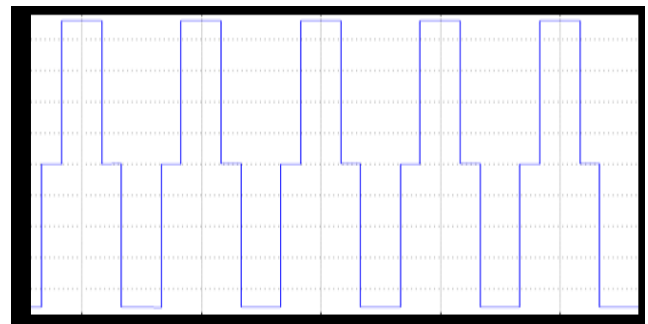


Fig. 5: Output voltage waveform of single phase inverter (period one operation)

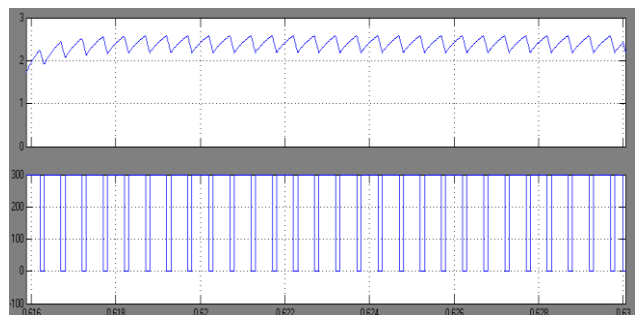


Fig.6: Enlarge view of output voltage and current waveform

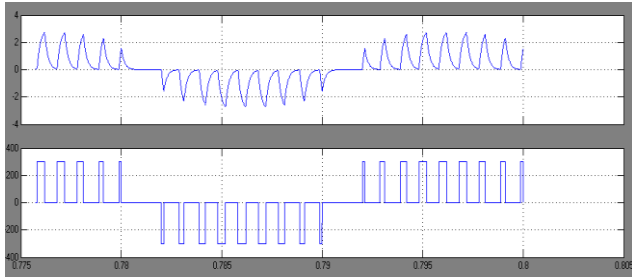


Fig.7: output voltage and current waveform

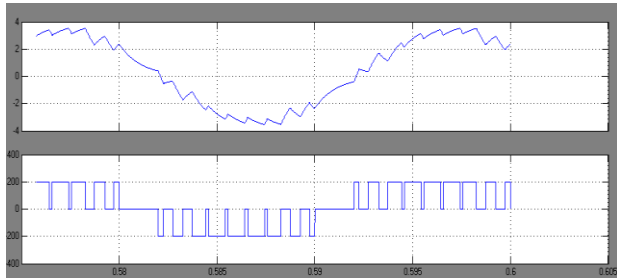


Fig 8: Enlarge view of output voltage and current waveform in unstable state

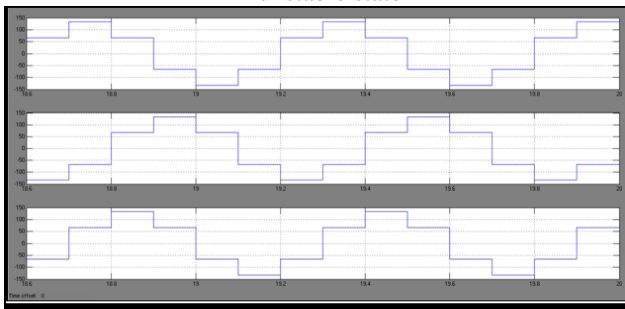


Fig. 9: Output Voltage Waveform of Power Bridge Inverter

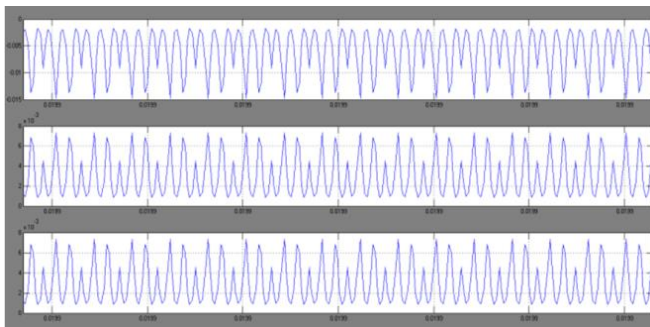


Fig. 10: Distorted load current after variation in Load

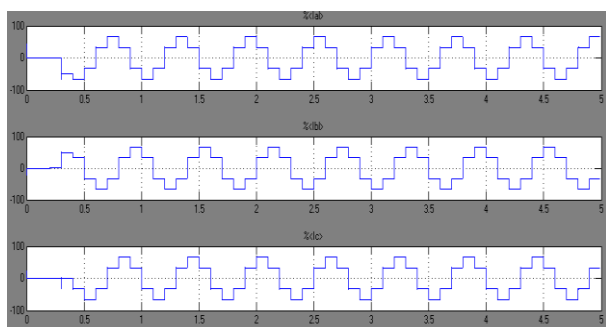


Fig. 11: Voltage waveform of Three Phase Inverter

Fig. 11 shows the voltage waveform of Three Phase Inverter when input voltage  $V_{in}=100\text{volt}$  ,  $R=100\ \Omega$  ,  $L=15\text{mh}$  , modulation index=0.35 frequency being 200Hz. the inverter operates in period one operation at this stage. Fig.12 and Fig.13

gives voltage waveform of Three phase Power H-Bridge inverter in unstable mode on input supply  $V_{in}=200\text{V}$  ,  $R=100\ \Omega$  ,  $L=10\text{mh}$  , modulation index=0.75 , carrier frequency=200Hz as the bifurcation parameter is varied. The same bifurcation structure is obtained when the bifurcation parameter are changed to original value. Fig. 15 Enlarge view of output voltage waveform in chaotic mode that shows inverter enter stable operation to unstable operating state.

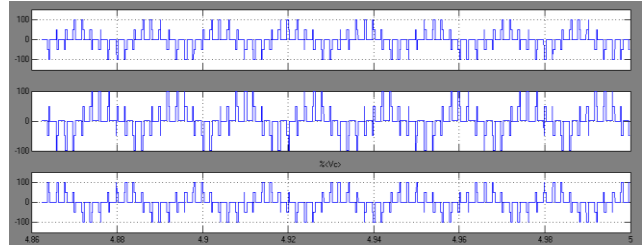


Fig. 12: Distorted voltage waveform of Three Phase Power H-Bridge Inverter

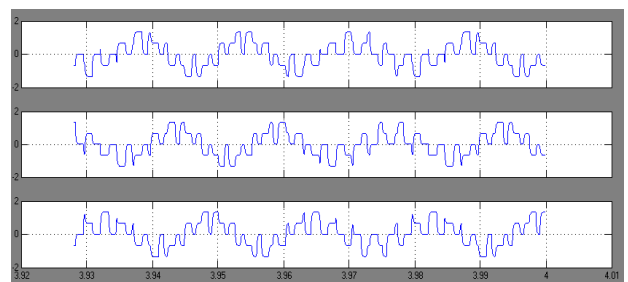


Fig. 13: Enlarge view of voltage waveform of Three Phase Power H-Bridge Inverter in unstable mode

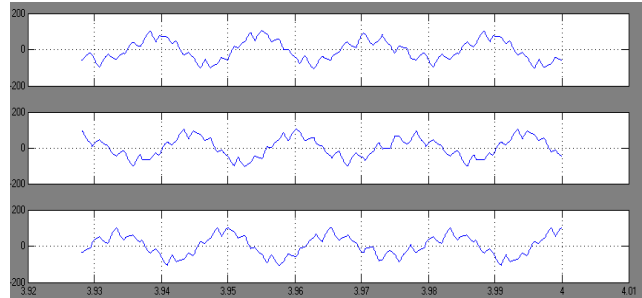


Fig. 14: Voltage waveform of Three Phase Power H-Bridge Inverter In chaotic mode

Fig. 7 shows an enlarged view of the Inductor current waveform when input voltage is 45V. Inductor current shows irregular pattern. It is bounded by maximum reference value and missed some clock pulses. Inductor current states do not repeat at the clock instant. Hence converter operates in chaotic mode

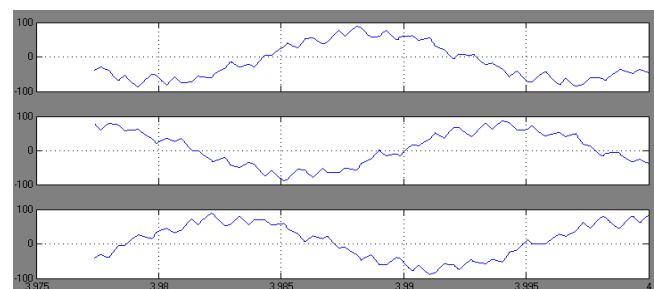


Fig. 15: Enlarge view of output voltage waveform in chaotic mode

#### IV. CASCADED NINE STEPPED INVERTER

The new development in the field of power electronics and microelectronics made it possible to reduce the magnitude of harmonics with multilevel inverters, in which the number of levels of the inverters are increased rather than increasing the size of the filters. The performance of multilevel inverters enhances as the number of levels of the inverter increases. The AC outputs of various full h-bridge inverters are connected in series such that the synthesized voltage waveform is the sum of the individual converter outputs.

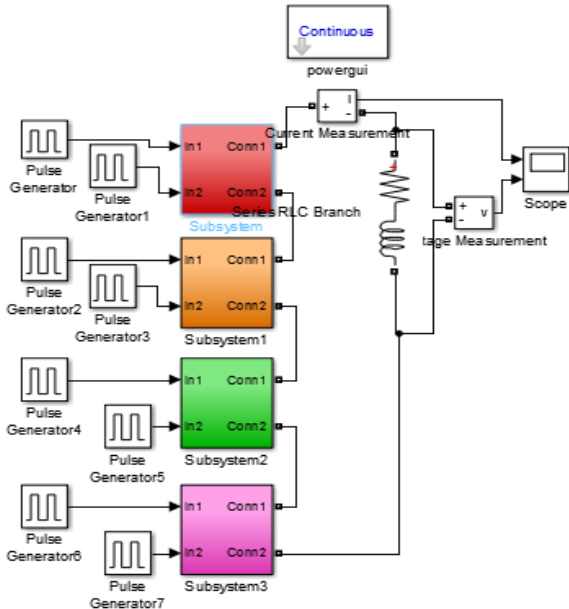


Fig. 16: Simulink model of nine stepped wave inverter

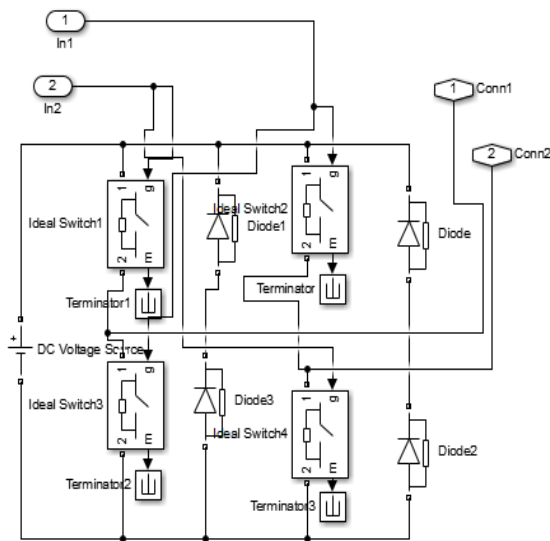


Fig. 17: Internal circuit for subsystem of nine stepped wave inverter

#### V. SIMULATION RESULTS OF STEPPED INVERTER

Simulation results are obtained by observing voltage and current waveforms at variable load and input supply voltage ( $V_{in}$ ) as bifurcation parameter. When the input supply voltage  $V_{in}$  varies from higher voltage to lower voltage, there is change in the dynamics. Fig.18 demonstrates the output voltage

waveform V2 recorded and current through when supply voltage waveform  $V_{in}$  was 3 volts and load have only  $R=10 \Omega$ , only. This waveform is periodic (period-1 operation). In Fig.20 shows the output voltage and current waveform in unstable state when  $R=10 \Omega$ ,  $L=10mH$  and input supply voltage  $V_{in} = 3V$ . Fig.22 to Fig 25 shows output voltage and current waveform in unstable state of nine stepped for the corresponding chaotic or strange attractor (graph between output voltage and current) has been drawn.

Inverter has transitioned by means of period doubling from periodic to chaotic operation. In short, power electronic circuits can exhibit nonlinear dynamics for example bifurcations, sub harmonic oscillations and chaos.

It is observed that the inductor current waveform during the switch on and switch off of the switch has “ringing” (fast damped oscillations) due to the presence of parasitic

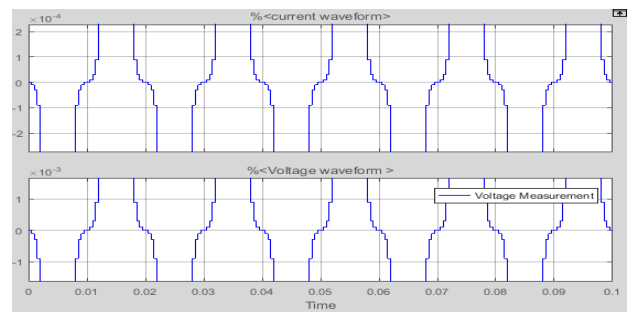


Fig. 18: Enlarge view of output voltage and current waveform in stable state (period-1)

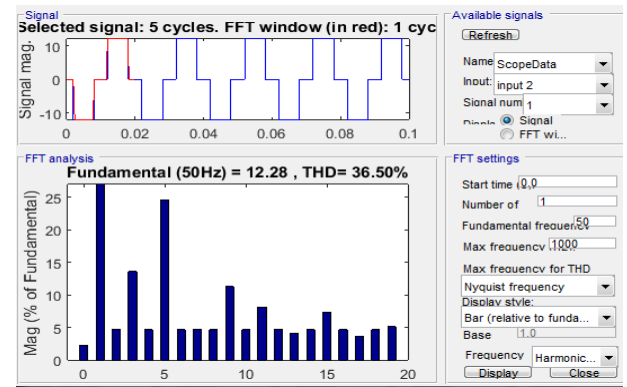


Fig. 19: FFT spectrum of nine stepped inverter for voltage ( $V_{in} = 3V$ ,  $R=10\Omega$ )

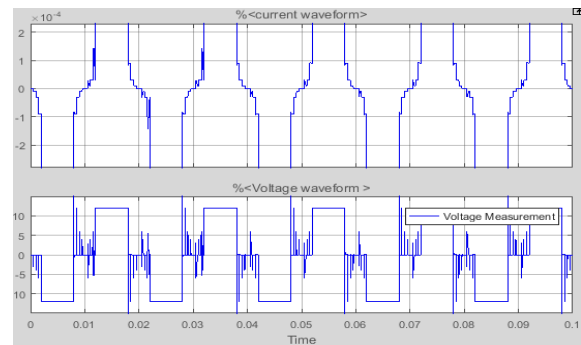


Fig. 20: Distorted output voltage and current waveform in unstable state

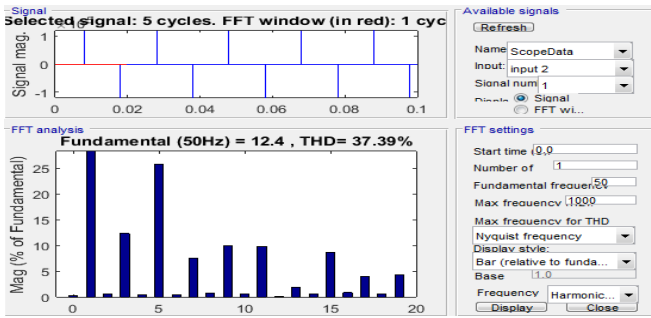


Fig. 21: FFT spectrum for voltage of nine stepped inverter for o/p voltage ( $V_{in} = 3V$ ,  $R=10\Omega$ ,  $L=10mH$ )

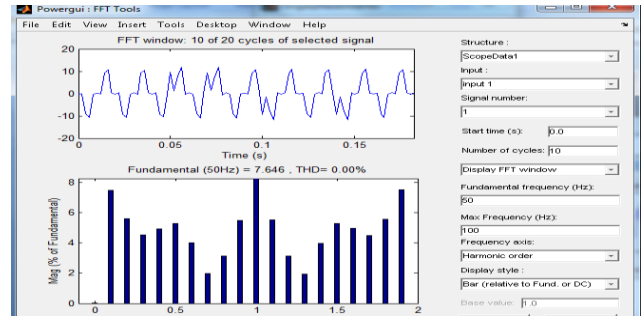


Fig. 26: FFT spectrum of nine stepped inverter for current

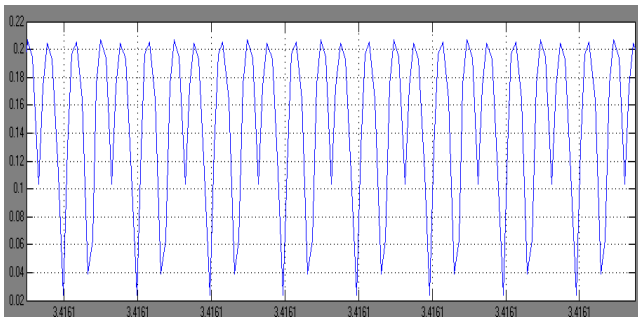


Fig. 22: Enlarge view of output current in period two operation

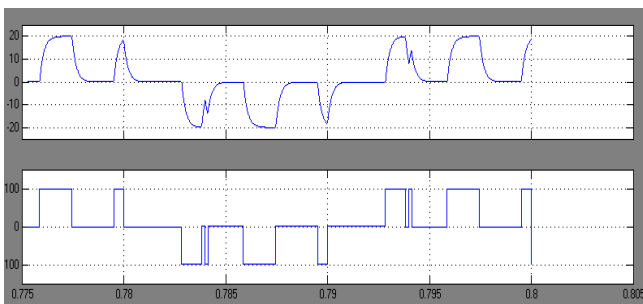


Fig. 23: output voltage and current waveform in unstable state

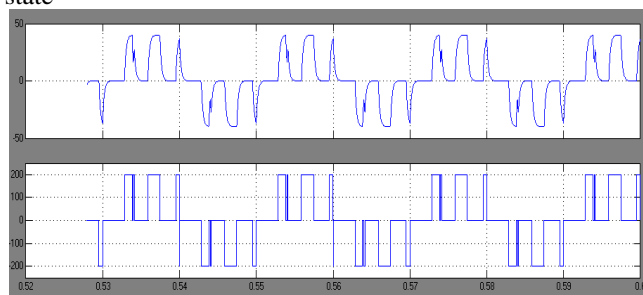


Fig. 24: Output voltage and current waveform in unstable state

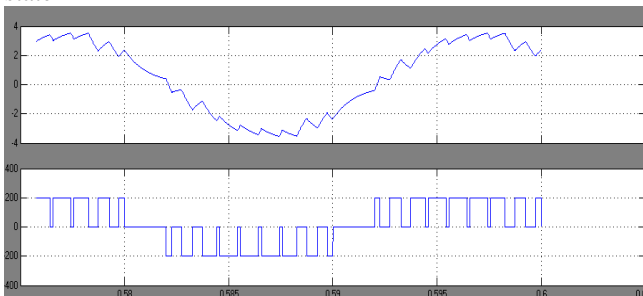


Fig. 25: Enlarge view of output voltage and current waveform in chaotic state

## VI. CONCLUSION

Bifurcation phenomena and nonlinear behavior investigated in the various types of dc to ac converter such as single, three and nine stepped wave dc to ac power converter. The inverter shows peculiar behavior as the bifurcation parameter like input voltage, to the inverter and nature of load is changed. Current, voltage waveforms and fast Fourier transform spectrum are obtained against various values of R,R-L load, which show that how inverters operates in period one, period two, higher periods and chaotic mode. The bifurcation pathway includes smooth period-doubling bifurcations as well as border collision bifurcations. Different values of the input voltage, load the orbit undergoes a bifurcation, and the system subsequently obtain to chaotic condition.

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