Optimizing Maintenance Strategies through Simulation Modeling: A Plant Simulation Approach.

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Abstract— This study explores the optimization of maintenance strategies using simulation modeling in Plant Simulation. It focuses on enhancing system reliability by examining key parameters such as availability, Mean Time To Repair, and Mean Time Between Failures, modeled through Negative Exponential and Weibull distributions. The interrelation of these parameters is leveraged to develop a dynamic maintenance model that addresses the limitations of traditional hazard rates. The study introduces a sawtooth hazard rate model for more realistic failure dynamics and evaluates the efficacy of maintenance strategies through comprehensive simulation experiments. The results indicate that incorporating maintenance significantly improves system reliability and operational efficiency, with detailed analysis provided through statistical tests and comparative assessments.

Index Terms— Event based simulation, Maintenance Strategies, System Reliability, Weibull and NegExp Functions

I. INTRODUCTION

A. Relevant Literature

The ScienceDirect database was used for the literature search. This limitation was primarily due to linguistic reasons, to ensure a focus on English-language texts.



Figure 1: Distribution of articles over the years

Based on the keywords Maintenance and DES, there is a continuously rising interest in the field. Around the year 2000, there were only a few publications, but the number steadily and strongly increased to approximately 2400. When reducing the keywords to Reliability Centered Maintenance and DES, the same trend is visible, although the total number of papers reduces to 8554. Further filtering with the keyword Hazard Rate brings the number of papers down to 437. If we further reduce the dataset by only considering research articles, we find 92 articles, of which

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87 are in the field of Engineering and 14 in Mathematics. Unfortunately, only 10 of these articles were directly accessible.







Figure 3: Classification of articles by subject area

- B. Key Takeaways
 - Methodologies: Articles employ various methodologies ranging from Monte Carlo simulations [1] and Markov models [2] to machine learning and statistical analysis [3].
 - Applications: Applications vary significantly, including aerospace [1], industrial machinery [4, 5], bridge infrastructure [6], offshore wind farms [7], and wave energy systems [8].
 - Focus: The focus ranges from specific component failure assessments [9] and maintenance optimization to broader topics like energy management and system resilience [10].

Each article addresses distinct aspects of reliability, maintenance, and optimization, tailored to specific applications and employing unique methodological approaches.

C. Aim and Motivation

- Methodologies The article employs simulation modeling in Plant Simulation to optimize maintenance strategies. It uses Weibull and Negative Exponential (NegExp) distributions to model Mean Time To Repair (MTTR) and Mean Time Between Failures (MTBF) [11]. It introduces a sawtooth hazard rate model for more realistic failure dynamics.
- Applications The focus is on enhancing system reliability and operational efficiency within a manufacturing station setup [12]. It evaluates the impact of different maintenance strategies under varied statistical distributions for failure intervals.

The article emphasizes dynamic maintenance modeling, realistic failure dynamics, and optimizing maintenance schedules. It addresses the limitations of traditional hazard rates and integrates a more nuanced approach to simulation modeling, considering human factors and operational conditions.

D. General Approach and Specific Ideas

- 1) General Approach
 - The article provides a comprehensive framework for optimizing maintenance strategies using advanced simulation models [13].
 - It incorporates statistical analyses and compares different distributions (NegExp vs. Weibull) to validate the effectiveness of maintenance strategies.
 - It includes detailed experimental design and parameters, focusing on practical applications and real-world scenarios [13].
- 2) Specific Ideas
 - The introduction of a sawtooth hazard rate model to represent dynamic failure rates.
 - Detailed analysis and optimization of Weibull distribution parameters to align with NegExp rate parameters.
 - A focus on the impact of preventive maintenance checks and system availability on performance and reliability.

In summary, this article thoroughly explores the specifics of simulation modeling and the optimization of maintenance strategies, offering a detailed and practical approach to enhancing system reliability and efficiency. This focus is more specialized compared to the broader range of reliability and maintenance topics discussed in the aforementioned articles.

E. Enhancing System Reliability through Simulation-Based Maintenance Strategies

In Plant Simulation, maintenance and failure dynamics are crucial elements that directly impact the overall effectiveness of operational processes. Key parameters in defining these dynamics include:

- Availability (Av): This reflects the proportion of time a system is in a functioning condition.
- Mean Time To Repair (MTTR): This measures the average time required to repair a machine or

system after a failure.

• Mean Time Between Failures (MTBF): This is the predicted elapsed time between inherent failures of a system during operation.

These parameters are interrelated through the formula

$$Av = \frac{MTBF}{MTTR + MTBF'}$$
(1)

which provides a basis for simulating the operational reliability of systems [15]. In Plant Simulation, the MTBF is typically modeled using a Negative Exponential (NegExp) distribution, assuming a random failure process with a constant hazard rate.

F. Connection Between Negative Exponential and Poisson Distributions

The connection between the Negative Exponential (NegExp) distribution and the Poisson distribution, particularly within the context of queuing theory, is a fundamental concept in the study of stochastic processes. These distributions are used to model and analyze systems involving random events over time [14].

1) Understanding the Negative Exponential Distribution

The Negative Exponential distribution is utilized to model the time between events in a continuous setting. It is defined by the rate parameter \lambda, representing the rate of occurrences (events per unit time). The probability density function (PDF) of the NegExp distribution is given by:

$$f(t) = \lambda e^{-\lambda t} \tag{2}$$

where $t, \lambda \ge 0$. This distribution is widely used to describe the time until the next event occurs in systems where events happen continuously and independently at a constant average rate [16].

2) Understanding the Poisson Distribution

The Poisson distribution models the number of discrete events in a fixed interval of time or space, assuming that these events occur with a constant mean rate and independently of the time since the last event. The probability mass function (PMF) of the Poisson distribution for a number of events k in a given time period t is:

$$P(k;\lambda t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$
(3)

where λ is the rate of event occurrences, t is the time interval, and k is the number of events.

3) Connection Between NegExp and Poisson

Distributions

The linkage between these distributions is highlighted by their shared parameters and foundational assumptions [17]:

- Rate Parameter λ: In both distributions, \lambda denotes the rate at which events occur. In the NegExp distribution, it describes the rate of time until the next event, while in the Poisson distribution, it quantifies the rate of occurrence of a number of events within a fixed period.
- Time Between Events: The NegExp distribution models the time between consecutive events in a Poisson process, demonstrating that if you have a Poisson process with rate λ, the time between each pair of consecutive events follows a NegExp distribution with the same λ.
- Modeling in Queuing Theory: These distributions

are commonly used together to model systems such as customer service centers, network traffic, or manufacturing processes. For example, the Poisson distribution might model the arrival of customers at a service point, while the NegExp distribution could model the service time required by each customer.

- 4) Focus and Application
 - NegExp: Focuses on the time between events.
 - Poisson: Focuses on the count of events within a specified time period.

The two are mathematically linked—specifically, if events follow a Poisson process (which they do if events occur randomly and independently at a constant average rate), then the time between events follows a Negative Exponential distribution. This intrinsic relationship allows for a comprehensive analysis and modeling of dynamic systems where understanding both the count of events and the timing between them is crucial.

II. CHALLENGES IN MODELING REALISTIC MAINTENANCE IMPACTS USING TRADITIONAL HAZARD RATES

The use of the Negative Exponential (NegExp) distribution in modeling Mean Time Between Failures (MTBF) is both a standard practice and a potential limitation. The constant hazard rate implied by this distribution suggests that failures are memoryless, meaning the probability of failure does not change over time or with the condition of the equipment [14]. This modeling approach does not account for:

- Wear and Tear: Equipment degradation over time which should realistically alter the failure rates.
- Maintenance Impact: The actual effects of maintenance activities on extending the life or restoring the condition of the equipment.

Consequently, maintenance strategies modeled in this framework might not accurately reflect their real-world impact on system reliability and efficiency. The primary challenge lies in integrating a more realistic approach to simulate how maintenance affects system performance and failure rates, especially in the context of equipment lifecycle.

A. Description of the Bathtub Curve

The bathtub curve is a fundamental concept in reliability engineering, depicting the failure rate of a product over its lifecycle [12, 14]. It is called the bathtub curve due to its shape, which resembles a cross-section of a bathtub. The bathtub curve is comprised of three distinct phases, each representing different failure characteristics over the lifecycle of a component or system:

- Break-in Phase
 - Description: This phase occurs at the beginning of the product's life. The failure rate is high but decreases over time as defective products fail and are removed from service.
 - •Causes: Failures are often due to manufacturing defects, material flaws, or design errors.
 - oStatistical Model: Failure rates typically

follow a decreasing Weibull distribution, where the shape parameter $\beta < 1$.

- Midlife Cycle Period
 - Description: This phase follows the infant mortality period and features a low and relatively constant failure rate, which makes it the most predictable and stable period.
 - •Causes: Failures are generally random and due to unforeseen operational or environmental conditions.
 - Statistical Model: The failure rate is often modeled using an exponential distribution, indicative of a constant failure rate where *λ* is typically low.

• Wear-Out Phase

- Description: Occurring towards the end of the product's life, this phase shows an increasing failure rate due to the aging and wear of components.
- Causes: Physical deterioration or obsolescence of components are typical reasons for the increase in failure rates.
- o Statistical Model: The increasing failure rate can be modeled by a Weibull distribution with a shape parameter $\beta > 1$, reflecting the wear-out characteristics.



Figure 4: Bathtub curve [Source: Own

The bathtub curve is extensively used in reliability engineering to design maintenance schedules, estimate product lifetimes, and make decisions about warranties and replacements. The bathtub curve provides a valuable tool for understanding and managing the reliability and maintenance requirements of various products throughout their lifecycle. By identifying and analyzing each phase, reliability engineers can implement more effective reliability and maintenance strategies, tailored to the specific needs of their products.

B. Midlife Cycle Modeling Restrictions

The effectiveness of the midlife cycle modeling in simulation environments is subject to several influencing factors beyond mere time metrics. These include the training and resources available to operational personnel and the material characteristics of the equipment alongside its operational load. Specific considerations include:

• Break-in Period Considerations: Contrary to traditional views that align the break-in period

solely with time, it is argued that this phase is more significantly impacted by how well the personnel are trained and equipped. Adequate training and access to necessary tools and materials can substantially minimize the duration of this initial phase. Thus, a focus on enhancing operational training and resource allocation during the break-in period is crucial.

Influence of Operational Conditions on Wear-Out: The latter stages of the equipment's lifecycle, often depicted as the wear-out phase in the bathtub curve, are heavily influenced not only by the inherent material properties but also by the operational practices employed. While excessive load may not alter the fundamental characteristics of this phase, it undoubtedly shortens the interval between the end of the break-in period and the onset of the wear-out phase. This underscores the importance of optimal machinery usage and the implementation of load management strategies to extend the effective life of the equipment.

These considerations necessitate a more nuanced approach to simulation modeling, where both human factors and operational tactics are integrated into the lifecycle analysis of machinery. By acknowledging and adjusting for these factors, simulations can more accurately reflect real-world conditions and offer more practical guidelines for maintenance and operation.

C. Incorporating Realistic Failure Dynamics into Plant Simulation

This paper aims to advance the modeling of maintenance in Plant Simulation by addressing the limitations of the NegExp distribution for MTBF in reflecting realistic maintenance outcomes. The focus will be on the following:

- Middle Life Cycle Modeling: Concentrating on the middle portion of the bathtub curve, which represents a period of constant failure rate typically observed after the initial break-in period but before the wear-out phase begins.
- Dynamic Failure Rate Adjustment: Developing a model that allows for adjustments to failure rates based on maintenance activities, moving beyond the assumption of a constant hazard rate to incorporate factors like improved condition and extended life due to preventive and corrective maintenance.

The goal is to create a more dynamic simulation model that more accurately reflects the experienced failure rates post-maintenance, providing a tool for more effective planning and execution of maintenance strategies.

III. CONCEPTUAL MODEL DEVELOPMENT

A. Sawtooth Hazard Rate Model

Conventionally, the hazard rate in the middle stage of the bathtub curve is depicted as a constant line, indicating a steady failure rate until the commencement of the wear-out phase. In contrast, our model introduces a dynamic representation, visualizing this period as a sequence of increasing hazard rates. These increments are periodically reset by maintenance actions, which lower the hazard rate to a new baseline. Each maintenance event essentially rejuvenates the system, mirroring the sawtooth pattern commonly seen in inventory management for reorder points and restocking cycles.

In our enhanced approach, the hazard rate (λ) starts at zero following a maintenance action and progressively climbs until the subsequent maintenance activity. The rate of increase may be linear or adopt a more complex form depending on the specific attributes of the equipment and the operating conditions. The flexibility of the Weibull distribution, with shape (β) and scale (η) factors, provides a robust framework for this model. Furthermore, the ability of the Weibull distribution to approximate the Negative Exponential distribution's behavior is leveraged.

In subsequent sections, we capitalize on the Weibull distribution's versatility. By setting the shape factor to 2, we construct a model that exhibits a linear increase in hazard rate. This feature renders the Weibull distribution particularly suitable for modeling a range of life behaviors influenced by the shape factor (β):

B. Detailed Analysis of Weibull and NegExp Distributions

The Weibull distribution is particularly noted for its flexibility in modeling various types of failure data, thanks to its adjustable parameters [18]. The general probability density function (PDF) of the Weibull distribution is described by the equation:

$$f(t) = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}$$
(4)

In this formula, γ represents the location parameter, β the shape factor, η the scale factor, and t is the time variable. During the interval between 0 and γ the system is free of failure. For reasons of simplicity the location parameter is set to zero and we can rewrite the probability density function to:

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$
(5)

The shape factor, β , influences the skewness of the distribution, allowing it to model a variety of life behaviors. For instance, a β less than 1 indicates a decreasing failure rate over time, typical in early product life (infant mortality phase), while a β greater than 1 suggests an increasing failure rate, characteristic of wear-out periods.

The hazard function $h(t) = \frac{f(t)}{1-F(t)}$, crucial for applications in reliability engineering, is derived directly from the PDF and is given by:

$$h(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta - 1} \tag{6}$$

This function indicates the instantaneous rate of failure at any given time t, assuming the component has survived up to that time. It is particularly useful in predicting the times of maintenance and replacement based on expected life distributions.

1) Comparison with NegExp Distribution

The Negative Exponential (NegExp) distribution is another critical tool in reliability engineering, commonly used to model the time between failures in a completely random process [17]. Its simplicity lies in a constant hazard rate, which is not dependent on time, making it ideally suited for electronic components and systems in their useful life phase. The PDF of the NegExp distribution is given by:

$$f(t) = \lambda e^{-\lambda t} \tag{7}$$

where λ is the rate parameter, indicating the number of failures per unit time and is constant over time. The NegExp distribution assumes that failures are memoryless, meaning that the probability of failure is the same regardless of how long the component has been in service. The hazard function:

$$h(t) = \lambda \tag{8}$$

2) Differentiating the Hazard Function with Respect to Time:

The derivative of the hazard function for the Negative Exponential (NegExp) distribution, given by $h(t) = \lambda$, with respect to t, is, as expected, zero for any t. This reflects the constant hazard rate characteristic of the NegExp distribution.

For the Weibull distribution, the hazard function is defined as $h(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1}$. Simplifying this expression, we obtain $h(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1}$. Taking the derivative of h(t) with respect to t yields:

$$\frac{d}{dt}h(t) = \frac{\beta(\beta-1)}{n^{\beta}}t^{\beta-2}$$
(9)

By setting $\beta = 2$, the derivative simplifies to: $\frac{d}{dt}h(t) = \frac{2}{\eta^2}$

$$\frac{d}{dt}h(t) = \frac{2}{\eta^2} \tag{10}$$

This result indicates that the rate of change of the hazard function is constant, showing a linear behavior when β is set to 2. This simplification highlights the unique properties of the Weibull distribution when modeling time-dependent failure rates.

When the shape factor β is set to 2, the Weibull hazard function simplifies significantly. The formula is then reduced to:

$$h(t) = \frac{2t}{\eta^2} \tag{11}$$

This linear relationship between the hazard rate and time provides a clear basis for the sawtooth model implementation.

To construct the sawtooth pattern, each segment of the model corresponds to the interval between two consecutive maintenance activities. By substituting t with the planned maintenance interval and setting \eta such that the hazard rate reaches a predefined maximum before maintenance, we can effectively model the reset of the hazard rate at each maintenance event. This approach ensures that the hazard rate is periodically reset, corresponding to the maintenance frequency, thereby creating a repeating sawtooth pattern which reflects periodic risk reduction due to maintenance activities.

C. Optimization of Weibull Distribution Parameters 1) Aligning Weibull Hazard Rate with NegExp Rate Parameter

The concept here is to synchronize the hazard rate of the Weibull distribution with the constant hazard rate λ of the NegExp distribution at a specific point in time. This point is chosen based on when the Weibull hazard rate equals the NegExp rate, ensuring that maintenance interventions are optimized to prevent an increasing risk of failure.

2) Mathematical Derivation and Selection of \eta



Figure 5: Sawtooth Hazard Rate Model

- a) NegExp Parameters
 - Mean Time Between Failures (MTBF) for NegExp is μ .
 - Hazard Rate (λ) for NegExp is $\frac{1}{4}$.
- *b*) Setting Weibull Hazard Equal to NegExp Hazard:
 - Weibull hazard rate at time t is given by

$$h(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta - 1} \tag{12}$$

• For $\beta = 2$, simplify this to

$$h(t) = \frac{2t}{\eta^2} \tag{13}$$

• Equating Weibull to NegExp at $t = \mu$:

Set
$$\frac{2\mu}{\eta^2} = \frac{1}{\mu}$$
 (14)

Solving for η yields

$$\eta = \sqrt{2}\mu \tag{15}$$

3) Implementing the Selection

By setting η to $\sqrt{2\mu}$, you're configuring the Weibull distribution to reset (i.e., schedule maintenance) once the hazard rate matches the constant rate of the NegExp distribution at the average failure time μ . This setting leverages the mean time to failure from the NegExp model inform the Weibull model, creating a coherent to maintenance strategy that activates just as the system's risk begins to exceed the average expected under constant hazard conditions.

- 4) Practical Application and Considerations
 - · Integration with Maintenance Policies: This method ensures that maintenance schedules are dynamically aligned with more actual operational risks, optimizing resource allocation and potentially reducing unnecessary maintenance actions.
 - Risk Management: By resetting the hazard rate at this critical point, you maintain control over the system's reliability, preventing the hazard rate from escalating beyond the expected average without intervention.
 - Predictive Maintenance: This approach can be further enhanced by integrating real-time monitoring data to adjust \mu and consequently \eta, allowing for an adaptive maintenance

schedule based on actual wear and environmental conditions.

5) Conclusion

The strategy to to set $\eta = \sqrt{2\mu}$ smartly ties the Weibull distribution's parameters to the fundamental characteristics of the NegExp distribution, optimizing maintenance intervals to a point where the failure risk begins to increase significantly. This method provides a solid statistical foundation for predictive and preventive maintenance programs, which can lead to improved reliability and operational efficiency.

IV. SIMULATION MODELS

A. Simulation Model Description



Figure 6: Comprehensive Maintenance and Failure Analysis Setup in Plant Simulation

This simulation model employs a systematic approach to evaluate the reliability and maintenance strategies of a manufacturing station within a single station setup. It is structured to analyze the influence of varying maintenance parameters under different statistical distributions for failure intervals. The core components of the model include:

- Source: Initiates the process by continuously supplying parts without constraints, representing a steady input flow.
- Processing Station: Central to the model, this station processes parts with a fixed duration of 10 seconds per part. The performance of this station is influenced by the predefined maintenance strategies and failure distributions.
- Drain: Collects processed parts, symbolizing the end of the production cycle

B. Experimental Design and Parameters

The experiment consists of 20 distinct runs divided into two main categories, each designed to scrutinize different aspects of maintenance strategies:

- 1. First 10 Experiments: Aimed at examining the station's behavior under varied maintenance schedules while keeping the frequency of failures approximately constant. This set alternates between:
 - Odd-numbered experiments employing a Negative Exponential (NegExp) distribution, focusing on a memoryless failure model.
 - Even-numbered experiments using a Weibull distribution with tailored parameters to ensure optimal maintenance timing.

The Mean Time To Repair (MTTR) is for both distributions adjusted across different availabilities and uses an Erlang distribution.

Explin	Availability	Distribution - Duration	MTTR	Sigma	Distribution - Interval	192	Beta	Eta
1	80%	Erlang	30:00.0000	21:12.8000	NegExp	2100:00.0000	0	-0.0000
2	50%	Driang	30:00.0000	21:12.0000	Theiball	-0.0000	2	2:49:42.0000
3	85%	Erlang	22:00.0000	15:33.4000	NegExp	2:04:40.0000	0	-0.0000
4	85%	Erlang	22:00.0000	15:33.4000	mitell	-0.0000	2	2:56:18.0000
5	90%	Erlang	14:00.0000	9:54.0000	NegExp	2:06:00.0000	0	-0.0000
6	90%	Erlang	14:00.0000	9:54.0000	weitell	-0.0000	2	2:58:11/0000
7	95%	Driang	7:00.0000	4:57.0000	NegExp	2:13:00.0000	0	-0.0000
8	95%	Erlang	7:00.0000	4:57.0000	meitall	-0.0000	2	3:06:05.0000
9	95%	Erlang	3:00.0000	2:07.3000	NegExp	2:27:00.0000	0	-0.0000
10	90%	Erlang	3:00.0000	2:07.3000	Webull	-0.0000	2	3:27:53.0000
11	80%	Erlang	30.00.0000	21:12.8000	NegExp	2100:00.0000	0	-0.0000
12	50%	Driang	30:00.0000	21:12.0000	Welbull	-0.0000	2	2:49:42.0000
13	85%	Erlang	30:00.0000	21:12.8000	NegExp	2:50:00.0000	0	-0.0000
14	85%	briang	30:00.0000	21:12.8000	mitell	-0.0000	2	4:00:25.0000
15	90%	Erlang	30:00.0000	21:12.9000	NegExp	4:30:00.0000	0	-0.0000
16	20%	Erlano	30.00.0000	21:12.8000	maited	-0.0000	2	6:21:50/0000
17	95%	Driang	30:00.0000	21:12.0000	NegExp	9:30:00.0000	0	-0.0000
18	95%	Erlang	30:00.0000	21:12.8000	meitall	-0.0000	2	13:26:06.0000
19	95%	Driang	30:00.0000	21:12.8000	NegExp	1:00:30:00.0000	0	-0.0000
20	96%	Edano	20:00.0000	21-12 0000	mahad	-0.0000	2	1-10-20-54 0300

Figure 7: Experimental Definitions and Parameters Overview

 Last 10 Experiments: These maintain a constant MTTR and vary only in terms of system availability, exploring the impact of operational uptime on performance.

C. Results of Experiments with the negative exponential distributions

Experiment	Total Throughput	Throughput per Day	Nr Of Failures	Fallure Rate
Exp_01 - Av: 00% - MTTR: 30:00.0000 (Interval by NegExp)	211009	7034	278	0.186
Exp_03 - Av: 85% - MTTR: 22:00.0000 (Interval by RegExp)	223377	7446	282	0.138
Exp_05 - Av: 90% - MTTR: 14:00.0000 (Interval by NegExp)	235355	7845	295	0.092
Exp_07 - Av: 95% - MTTR: 7:00.0000 [Interval by NegDxp]	247354	8245	293	0.046
Exp_09 - Avi 98% - MTTRi 3:00.0000 [Interval by NegRxp]	254420	0401	275	0.019
Exp_11 - Av: 00% - MTTR: 30:00.0000 [Interval by NegExp]	211009	7034	278	0.185
Exp_53 - Av: 85% - MTTR: 30:00.0000 [Interval by NegExp]	222185	7405	210	0.143
Exp_15 - Avi 90% - MTTR: 30:00.0000 (Srterval by RegExp)	234035	7868	139	0.089
Exp_17 - Av: 95% - MTTR: 30:00.0000 [Interval by NegExp]	247227	8241	70	0.046
Exp_19 - Avi 98% - MTTRI 30:00.0000 Disterval by Realize)	254624	0400	27	0.019

Figure 8: Summary Table of Experiments with Negative Exponential Distribution



Figure 9: Graphical Summary of Experimental Results with Negative Exponential Distribution

Metric	Exp_01 - Av: 80% - MTTR: 30:00.0000 [Interval by NegExp]	Exp_03 - Av: 85% - MTTR: 22:00.0000 [Interval by NegExp]	Exp_05 - Ax: 90% - MTTR: 14:00.0000 [Interval by NegExp]	Exp_07 - An: 95% - MTTR: 7:00.0000 [Interval by NegExp]	Exp_09 - Av: 98% - MTTR: 3:00.0000 [Interval by NegExp]	Exp_11 - Av: 80% - MTTR: 30:00.0000 [Internal by NegExp]	Exp_13 - Av: 85% - MTTR: 30:00.0000 [Interval by NegExp]	Exp_15 - Av: 90% - MTTR: 30:00.0000 [Interval by NegExp]	Exp_17 - Av: 95% - MTTR: 30:00.0000 [Interval by NegExp]	Exp_19 - Av: 98% - MTTR: 30:00.0000 [Interval by NegExp]
count	292	295	303	303	292	292	221	135	76	27
mean	122.9	128.5	130.9	137.8	148.6	122.9	171.4	281	541.3	1440.6
stó	54.3	55.9	56	59.3	66.9	54.3	81.5	126.9	280.1	794.3
min	12.4	25.3	27.9	29.5	15.2	12.4	14.9	90.6	79.4	114.4
25%	83.7	87.9	91.1	95.5	100.7	83.7	109.2	191.1	320.7	883.9
50%	113.9	119.2	123.1	130	138.2	113.9	156.2	255.9	508.6	1174.5
75%	155.8	162.9	165.6	174.3	185.5	155.8	223.7	359.8	743.4	2144.6
199.2V	776.4	297.1	200.2	205.2	120.5	226.4	201.5	671.0	1212.7	2207.2

Figure 10: Descriptive Statistics of Experiments with Negative Exponential Distribution

Exp_01 - AV: 80% - MTTR: 30:00.0000 [Interval by NegExp]	0.468	
Exp_03 - Av: 85% - HTTR: 22:00.0000 [Interval by NegExp]	0.467	
Exp_05 - Av: 90% - MTTR: 14:00.0000 [Interval by NegExp]	0.458	
Exp_07 - Av: 95% - HTTR: 7:00.0000 [Interval by NegExp]	0.435	
Exp_09 - Av: 98% - HTTR: 3:00.0000 [Interval by NepExp]	0.404	
Exp_11 - Av: 80% - HTTR: 30:00.0000 [Interval by NegExp]	0.488	
Exp_13 - Av: 85% - HTTR: 30:00.0000 [Interval by NegExp]	0.35	
Exp_15 - Av: 90% - MTTR: 30:00.0000 [Interval by NegExp]	0.213	
Exp_17 - Av: 95% - HTTR: 30:00.0000 [Interval by NegExp]	0.111	
Fxn: 19 - Av: 98% - HTTR: 20:00.0000 [Interval by NenFxn]	0.047	

Figure 11: Fitting Results of MTBF to Negative Exponential Distribution

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D. Results of Experiments with the Weibull distributions

Experiment	Total Throughput	Throughput per Day	Nr Of Failures	Fallure Rate
Exp_02 - Av: 80% - MTTR: 30:00.0000 [Interval by Webuil]	217630	7254	237	0.16
Exp_04 - Av: 85% - MTTR: 22:00.0000 [Interval by Weibuil]	228519	7617	229	0.119
Exp_06 - An1 90% - MTTR: 14:00.0000 [Driterval by Weibuil]	238943	7965	247	0.078
Exp_08 - An: 95% - MTTR: 7:00.0000 (Interval by Weibull)	249226	8308	243	0.038
Exp_10 - An: 93% - MTTR: 3:00.0000 [Interval by Weibul]	255227	8508	226	0.015
Exp_12 - Avi 80% - MTTRi 30:00.0000 [Interval by Webuil]	217630	7254	227	0.16
Exp_34 - An: 85% - MTTR: 30:00.0000 [Interval by Weibuil]	229093	7636	176	0.116
Exp_16 - An: 90% - MTTR: 30:00.0000 [Interval by Weibull]	239914	7997	118	0.074
Exp_18 - Av: 95% - MTTR: 30:00.0000 [Interval by Webuil]	249333	8311	\$7	0.030
Exp.,20 - Avi 98% - MTTR: 30:00.0000	255614	8520	22	0.014

Figure 12: Summary Table of Experiments with Weibull Dis- tribution



Figure 14: Descriptive Statistics of Experiments with Weibull Distribution

Ratio	(unlide (Referenced is feared)	
Dag, 32 - Arc 2016 - HTTE: 30-30. 2000 [Internative Websel]	1.00	
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Figure 15: Fitting Results of MTBF to Negetive Exponential Distribution

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(14,11) to 10%, 10%, 21,00,000 (16,14) to 16(14)	1.407	,		12
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14,01-14,105-1070,110.000 (research holds)	1.404	,		1.107
14,11-14,10%,10%,10.00.000 (results head)	1.400			1.100
tas, 12 - Arc 1076, 19776, 2018, 2018	1.0			1.00
ta, il active with local and	8.217	,		1.07
THE R. LEWIS CO., LANSING MICH.	8.000	,		18.10
NALT ALL MALE AND A DESIGNATION OF THE OWNER	1.00	1		16.267

Figure 16: Fitting Results of MTBF to the Weibull Distribution

E. Statistical Analysis (Independent Samples t-test)

The results of the t-tests for comparing the Negative Exponential (even-numbered experiments) and Weibull (odd-numbered experiments) distributions across the four metrics are as follows:

- 1) Total Throughput and Throughput per Day
 - Null Hypothesis (H₀): There is no significant difference in Total Throughput between the Negative Exponential and Weibull distributions. H₀: μ_{Negative Exponential} = μ_{Weibull}
 - Alternative Hypothesis (*H*₁): There is a significant difference in Total Throughput between the Negative Exponential and Weibull distributions.

 $H_1: \mu_{\text{Negative Exponential}} \neq \mu_{\text{Weibull}}$

- a) Results:
 - t-statistic: -0.552
 - p-value: 0.587
- 2) Number of Failures
 - Null Hypothesis (H₀): There is no significant difference in the Number of Failures between the Negative Exponential and Weibull distributions. H₀: μ_{Negative Exponential} = μ_{Weibull}
 - Alternative Hypothesis (H₁): There is a significant difference in the Number of Failures between the Negative Exponential and Weibull distributions. H₁:µ_{Negative Exponential} ≠ µ_{Weibull}
 - a) Results:
 - t-statistic: 0.838
 - p-value: 0.413
- *3)* Failure Rate
 - Null Hypothesis (H₀): There is no significant difference in the Failure Rate between the Negative Exponential and Weibull distributions. H₀: μ_{Negative Exponential} = μ_{Weibull}
 - Alternative Hypothesis (H₁): There is a significant difference in the Failure Rate between the Negative Exponential and Weibull distributions. H₁:µ_{Negative Exponential} ≠ µ_{Weibull}
 - a) Results:
 - t-statistic: 0.552
 - p-value: 0.588

4) Summary

In our analysis, we used the independent samples t-test to compare the performance metrics (Total Throughput, TpD, Number of Failures, and Failure Rate) between two groups: those using the Negative Exponential distribution and those using the Weibull distribution. We calculated the t-statistics and p-values for each metric. The p-values were all greater than 0.05, indicating that the differences in means between the two groups were not statistically significant. Thus, while the Weibull distribution appears to perform better in terms of raw metrics, the improvement is not statistically significant enough to definitively state that the Weibull yields different results.

F. Maintenance Implementation

The implementation of maintenance in the simulation model is managed using a Generator object. The Generator object controls the timing and duration of maintenance activities, ensuring they are accurately represented within the system's operational timeline.

1) Interval and Duration:

• Interval: This parameter defines the time between two activations of the Interval control, effectively setting the period from the end of one maintenance event to the start of the next maintenance event.

- Duration: This parameter defines the time span between the activation of the Interval control and the activation of the Duration control, which marks the end of the maintenance period.
- 2) Start of Maintenance:

At the start of a maintenance event, the following actions are performed:

- If a failure is active, it is deactivated, and the maintenance process is initiated.
- The failure profile itself is also deactivated at the start of maintenance and reactivated at the end of the maintenance period.

3) End of Maintenance:

At the end of a maintenance event, the following actions are performed:

- The start time of the failure profile is set to the current time.
- The stop time of the failure profile is set to the start of the next maintenance event, ensuring that no failures are initiated during the maintenance period.
- The failure profile is reactivated.

4) Initialization:

Both the failure profile and the Maintenance Generator are set up at the initiation of each simulation run. This setup ensures that the timing and effects of maintenance are consistently applied throughout the simulation, allowing for accurate modeling of the system's reliability and maintenance dynamics.

G. Experimental Design and Parameters

The experiment consists of 20 distinct runs divided into two main categories, each designed to scrutinize different aspects of maintenance strategies:

1) First 10 Experiments

The first 10 experiments aim to examine the station's behavior under varied maintenance schedules while keeping the frequency of failures approximately constant, as defined by the Negative Exponential (NegExp) distributions. This set alternates between two types of experiments:

- Odd-numbered experiments: These employ a Negative Exponential (NegExp) distribution, focusing on a memoryless failure model without any maintenance.
- Even-numbered experiments: These use a Weibull distribution with tailored parameters to ensure optimal maintenance timing, including the implementation of a maintenance generator.

For both distributions, the Mean Time To Repair (MTTR) is adjusted across different availabilities and further uses the Erlang distribution. The key parameters for these experiments are:

• Maintenance Duration: Set to 0.5 of the mean (\mu) of the Erlang distribution and is kept constant.

• Maintenance Interval: Set to 0.5 of the mean (\mu) of the NegExp distribution.

EqNr	Availability	MITTR	Interval Time	Repair Time	
2	80%	30:00.0000	2:00:00.0000	15:00.0000	
4	85%	22:00.0000	2:04:40.0000	7:46.7000	
6	90%	14:00.0000	2:05:00.0000	4:57.0000	
8	95%	7:00.0000	2:13:00.0000	2:28.5000	
10	96%	3:00.0000	2:27:00.0000	1:03.6500	
12	80%	30:00.0000	1:20:00.0000	10:00.0000	
14	85%	30:00.0000	1:53:20.0000	10:00.0000	
16	90%	30:00.0000	3:00:00.0000	10:00.0000	
18	95%	30:00.0000	6:20:00.0000	10:00.0000	
20	46%	30:00.0000	16:20:00.0000	10:00.0000	

Figure 17: Summary of the additional Experimental Maintenance Definitions on the Weibull Distributions



Figure 18: Summary Table of Experiments with Weibull Distribution extended with Maintenance Interval and Duration

2) Last 10 Experiments:

The last 10 experiments maintain a constant MTTR and vary only in terms of system availability, exploring the impact of operational uptime on performance. In these experiments, the maintenance interval is adjusted to 0.3 of the mean (\mu) of the NegExp distribution, rather than 0.5. This allows for a detailed examination of how different maintenance intervals affect system performance and reliability.

- MTTR: Constant across all experiments.
- System Availability: Varied to explore its impact on performance.
- Maintenance Interval and Duration: Set to 0.3 of the mean (\mu) of the NegExp distribution and the Erlang distribution accordingly.
- 3) Results after activating Maintenance Jobs

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Metric	Exp_02 - 80% 30 15	Exp_04 - 85% 22 11	Exp_06 - 90% 14 7	Exp_08 - 95%[7]2.5	Exp_10 - 98%[2]1.5	Exp_12 - 80% 30 10	Exp_14 - 85% 30 10	Exp_16 - 90% 30 10	Exp_18 - 95% 30 10	Exp_20 - 98%6[20]
count	108	162	159	142	138	112	86	50	27	7
mean	246.2	258.5	264.1	288.6	308.4	378.4	493.5	739.2	1501.1	3738.5
std	110	117.6	106.7	117.4	115.1	105	231.4	305.6	690.2	1399.3
min	22.6	18.6	67.2	117.5	88.3	106	85.2	162	477.5	1830.6
25%	174.7	177.7	186.1	206.3	233.4	264.2	340.3	521.9	1070.3	2949
50%	237.5	250.6	255	262.5	293.3	361.1	474.6	722.4	1420.7	3536.3
75%	306.3	314.9	331.7	351.3	385.3	471.2	599.4	889.1	1819.4	4417
max	732.3	051.5	613.4	667.1	686.7	1191.1	1445	1785	3648.3	6068.8

Figure 20: Descriptive Statistics of Experiments with Weibull Intervals and Maintenance active

Name	Lambda [Referenced to hours]
Exp_02 - Av: 80% - MTTR: 30:00.0000 [Maintenace Duration: 15:00]	0.2437363854
Exp_04 - Av: 85% - MTTR: 22:00.0000 [Maintenace Duration: 11:00]	0.2321262048
Exp_06 - Av: 90% - MTTR: 14:00.0000 [Maintenace Duration: 7:00]	0.2272254669
Exp_08 - Av: 95% - MTTR: 7:00.0000 [Maintenace Duration: 3:30]	0.2078809125
Exp_10 - Av: 98% - MTTR: 3:00.0000 [Maintenace Duration: 1:30]	0.1945462079
Exp_12 - Av: 80% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	0.1585603678
Exp_14 - Av: 85% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	0.1215907312
Exp_16 - Av: 90% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	0.0811685121
Exp_18 - Av: 95% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	0.039971851
Exp_20 - Av: 98% - MTTR: 30:00.0000 [Maintenace	0.0160490888

Figure 21: Fitting Results of MTBF to Negative Exponential Distribution (Maintenance active)

Name	Weibull Shape (beta)	Weibull Loc	Welbull Scale (eta) [Referenced to hours]
Exp_02 - Av: 80% - MTTR: 30:00.0000 [Maintenace Duration: 15:00]	2	0	4.492
Exp_04 - Av: 85% - MTTR: 22:00.0000 [Maintenace Duration: 11:00]	2	0	4.73
Exp_06 - Av: 90% - MTTR: 14:00.0000 [Maintenace Duration: 7:00]	2	0	4.744
Exp_08 - Av: 95% - MTTR: 7:00.0000 [Maintenace Duration: 3:30]	2	0	5.191
Exp_10 - Av: 98% - MTTR: 3:00.0000 [Maintenace Duration: 1:30]	2	0	5.484
Exp_12 - Av: 80% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	2	0	6.875
Exp_14 - Av: 85% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	2	0	9.074
Exp_16 - Av: 90% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	2	0	13.314
Exp_18 - Av: 95% - MTTR: 30:00.0000 [Maintenace Duration: 10:00]	2	0	27.447
Exp_20 - Av: 98% - MTTR: 30:00.0000 [Maintenace Duration:	2	0	65.944

Figure 22: Fitting Results of MTBF to Weibull Distribution (Maintenance active



Figure 19: Graphical Summary of Experimental Results with Weibull Distribution and Maintenance active

V. DISCUSSION OF RESULTS



Figure 23: Experiments with Constant frequency of failures and changing Availability



Figure 24: Experiments with Constant MTTR for failures and changing Availability

A. Without Maintenance (Left Side)

1) Overview

The histograms show the distribution of the time between failures and the time to repair. Generally, the distributions appear skewed, with a higher frequency of shorter intervals and repairs.

2) Failure Frequency

This suggests that failures occur relatively frequently, and repairs are often completed in shorter durations.

3) Variation in Time Between Failures

There is also a noticeable variation in the time between failures, indicating that the system's reliability is inconsistent over time.

B. With Maintenance (Right Side)

1) Impact of Maintenance Checks

The introduction of maintenance checks changes the distribution of time between failures. The histograms indicate that maintenance checks at fixed intervals tend to increase the time between failures, as preventive maintenance helps to mitigate unexpected failures.

2) Duration of Maintenance Checks

The fixed duration for maintenance checks is typically shorter than the repair times, as shown in the histograms. This is logical, as preventive maintenance is usually less time-consuming than corrective maintenance.

3) Decrease in Short Repair Times

The frequency of short repair times decreases, suggesting that regular maintenance helps to avoid certain types of failures that would otherwise require immediate repair.

C. Comparison Between Experiments

1) Experiment 1 (80% Availability, MTTR Without Maintenance)

The system has frequent failures with shorter repair times. The introduction of maintenance checks increases the time between failures and stabilizes the system's reliability.

2) Experiment 2 (85% Availability, MTTR Without Maintenance)

Similar trends as Experiment 1, but with slightly less frequent failures. Maintenance checks continue to improve the time between failures.

3) Higher Availability Experiments (90%, 95%, 98%)

With increasing availability, the time between failures without maintenance increases. Maintenance checks further extend the time between failures, demonstrating their effectiveness. The higher the availability, the more significant the impact of maintenance checks on improving system reliability.

D. Insights

1) Preventive Maintenance

Regular maintenance checks are crucial for improving the system's reliability and reducing the frequency of failures.

2) System Availability

systems benefit more Higher availability from maintenance checks as they already have a lower frequency of failures.

3) Consistency

Maintenance checks help in making the time between ^[5] failures more consistent, reducing the unpredictability of system downtimes.

E. Summarv

In summary, the data and histograms collectively highlight the importance of maintenance checks in enhancing system performance and reliability. The [7] M. Liu, J. Qin, D.-G. Lu, W.-H. Zhang, J.-S. Zhu, and M. H. Faber, experiments demonstrate that while systems with higher inherent availability perform better, preventive maintenance universally contributes to extending the operational periods between failures and reducing downtime. Furthermore, [8] Z. Shahroozi, M. Göteman, E. Nilsson, and J. Engström, "Environmental regular maintenance checks are crucial in enhancing the

consistency and predictability of system downtimes. They reduce the variability in the time between failures, making it easier to manage and predict system performance. This predictability not only improves the overall reliability of the system but also ensures that inter-operation buffers are used effectively, minimizing disruptions and maintaining a smooth operational flow.

VI. CONCLUSION

The research presented demonstrates the critical role of dynamic maintenance strategies in enhancing the reliability and efficiency of operational systems. By integrating more realistic failure dynamics through the sawtooth hazard rate model and leveraging the flexibility of the Weibull distribution, the study successfully addresses the limitations of the traditional Negative Exponential distribution. The simulation experiments validate that regular maintenance checks significantly extend the time between failures, reduce downtime, and stabilize system performance. Statistical analyses confirm that while higher availability systems inherently perform better, preventive maintenance universally enhances system reliability. This research underscores the importance of adaptive maintenance schedules based on real-time monitoring and operational conditions, offering a robust framework for predictive and preventive maintenance programs. The findings provide valuable insights for optimizing maintenance strategies, thereby contributing to more efficient and reliable industrial operations.

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